CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, July/August 2021
Additional Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions.***

1 a. Express $\frac{(3+i)(1-3i)}{(2+i)}$ in the form x + iy. **(06 Marks)**

b. Find the modulus and amplitude of the complex number $1 + \cos\alpha + i\sin\alpha$. **(05 Marks)**

c. If $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$, then find $\vec{a} \times (\vec{b} \times \vec{c})$. **(05 Marks)**

2 a. Prove that $\left[\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right]^n = \cos n\theta + i\sin n\theta$. **(06 Marks)**

b. Find the cube root of $1 + i\sqrt{3}$. **(05 Marks)**

c. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar. **(05 Marks)**

3 a. Find the n^{th} derivative of $e^{ax}\sin(bx + c)$. **(06 Marks)**

b. With usual notations prove that $\tan\phi = r \cdot \frac{d\theta}{dr}$. **(05 Marks)**

c. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. **(05 Marks)**

4 a. Find the n^{th} derivative of $\frac{x}{(x-2)(x-3)}$. **(06 Marks)**

b. Find the angle between the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$. **(05 Marks)**

c. Given $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(05 Marks)**

5 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi/16} \cos^5(8x)\sin^6(16x)\,dx$. **(05 Marks)**

c. Evaluate $\int_1^2\int_1^3 x y^2 \,dx\,dy$. **(05 Marks)**

6 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\,d\theta$. **(05 Marks)**

c. Evaluate $\int_{-3}^{3}\int_0^1\int_1^2 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

7 a. Find velocity and acceleration of a particle moving along the curve $\vec{r} = e^{-2t}\,\hat{i} + 2\cos 5t\,\hat{j} + 5\sin t\,\hat{k}$ at anytime t. Find their magnitudes at t = 0. (06 Marks)

b. If $\phi = x^3 + y^3 + z^3 - 3xyz$ find $\nabla\phi$ at (1, -1, 2). (05 Marks)

c. Show that $\vec{F} = (x + 3y)\,\hat{i} + (y - 3z)\,\hat{j} + (x - 2z)\,\hat{k}$ is Solenoidal. (05 Marks)

8 a. Find the unit tangent vector of the space curve $\vec{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$. (06 Marks)

b. If $\vec{F} = x^2y\,\hat{i} + yz^2\,\hat{j} + zx^2\,\hat{k}$, then find div (curl $\vec{F}$). (05 Marks)

c. Find the constants a, b and c such that the vector $\vec{F} = (x + y + az)\,\hat{i} + (x + cy + 2z)\,\hat{j} + (bx + 2y - z)\,\hat{k}$ is irrotational. (05 Marks)

9 a. Solve $\dfrac{dy}{dx} = 1 + y/x + \left(y/x\right)^2$. (06 Marks)

b. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. (05 Marks)

c. Solve $\dfrac{dy}{dx} = \dfrac{x^2 - 2xy}{x^2 - \sin y}$. (05 Marks)

10 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$. (06 Marks)

b. Solve $(1 + xy)y\,dx + (1 - xy)\,x\,dy = 0$. (05 Marks)

c. Solve $x\dfrac{dy}{dx} + y = x^3y^6$. (05 Marks)

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